Ultra-High Energy Cosmic Rays from Galactic Supernovae

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Abstract

A simple phenomenological formula is developed relating one ion energy, the collision energy of an ion and a nucleus at rest in the Earth's atmosphere, to another ion energy, the trajectory energy of the ion before the collision. The resulting formula realizes the possibility that ultra-high energy cosmic rays are products of the supernovae in our Galaxy while recognizing that terrestrial experiments have not yet detected any effect. If the collision-trajectory energy difference is one or two percent for 1 TeV protons at the Tevatron, then a readily apparent difference should occur with a 7 TeV trajectory at the LHC.

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1   Introduction

Before colliding with a nucleus in the Earth's atmosphere, a cosmic ray ion had a trajectory through the Galaxy. If the collision energy and trajectory energy can differ, then an ion accelerated in a supernova-remnant shockwave traveling a low energy trajectory can deliver a much higher energy upon collision in the atmosphere. However, as yet, there are no reports of any such effects in terrestrial accelerator labs.

The conjecture is that *the collision energy and the trajectory energy differ by a small amount at a low energy and differ by a large amount at a high energy.*

The 'trajectory energy' E_T can be determined for a specific ion by measuring time-of-flight over a given distance or by measuring the orbit radius in a known magnetic field. Let us concede that the ions accelerated in SNRs bend in magnetic fields along trajectories that are determined by standard gravitation and electrodynamics.

Measurements of the 'collision energy' E_C of a primary cosmic ray occur indirectly by observing the secondaries in the Earth's atmosphere. Each secondary has a much lower energy than the primary. At these low energies, our conjecture requires that each secondary's collision and trajectory energies are nearly equal. It follows that the sum of the secondaries' trajectory energies equals the collision energy E_C of the primary.

At any energy, the trajectory energy E_T can be distinguished from the collision energy E_C by the way the energy is measured.

The collision and trajectory differ by the forces involved. The trajectory occurs in the long range electromagnetic and gravitational fields of distant sources. The collision of an ion with a nucleus in the Earth's atmosphere involves much stronger electromagnetic and somewhat stronger gravitational fields as well as intense short range forces. If the strong force is relevant, then the formula obtained below may be obeyed by hadrons and not by leptons. If gravitation is important, then collisions of ions and nuclei in deep space may not obey the formula obeyed by ion-nucleus collisions on Earth.

The formula ramps up the dependence of collision energy on trajectory energy from near equality at 1 TeV = 10^{12} eV to a multiplier of 10^5 for 10^{15} eV trajectories. The steepness implies that if 1 TeV protons at the Tevatron at Fermilab [1] are near threshold, then an effect should be apparent for 7 TeV protons at the Large Hadron Collider (LHC) at CERN.[2] If real, the effect would be remarkable because a design-estimated 7 TeV proton should deliver more energy, up to say 14 TeV, in collision, a gain not a loss.

2   The Two Energies Formula

Our Galaxy's supernovae remnants (SNRs) have shock waves that are thought to accelerate particles to energies as high as 10^{15} eV.[3] On Earth, cosmic rays deposit as much as 10^{20} eV into the atmosphere.[4, 5] If the highest energy cosmic rays where originally accelerated in the Galaxy's SNRs, then there should be some way to account for the factor of 10^5, the ratio of the highest energy cosmic rays to the highest energy SNR-accelerated particles. Since 10^5 is a large factor, we make a rough order-of-magnitude calculation involving data mostly rounded to integral powers of ten.

By conjecture, the collision and trajectory energies of an ion differ for large E_T and agree for low E_T. Consider the following formula for the collision energy E_C as a function of the trajectory energy E_T,

$$E_C = E_T + \frac{E_T^n}{E_0^{n-1}} \quad , \tag{1}$$

where n is a positive integer and E_0 is the 'threshold energy'. When E_T is much larger than the threshold energy E_0, $E_T \gg E_0$, we have $E_C \gg E_T$. At low energies, when E_T is much less than the threshold energy E_0, $E_T \ll E_0$, we have $E_C \approx E_T$. We seek n and E_0.

Consider the trajectories of protons. One expects that 10^{15} eV protons travel in tight spirals along the 10^{-10} T Galactic magnetic field lines.[6] However, 10^{20} eV protons do not spiral tightly along 10^{-10} T field lines. Instead of the one parsec radius for 10^{15} eV, a 10^{20} eV proton would 'orbit' 10^{-10} T field lines with a super-galactic radius of 10^5 parsecs and quickly leave the Galaxy. One infers that if the highest energy ions from SNRs contribute significantly to the population of ultra-high energy cosmic rays seen on Earth, then they move in the Galaxy along $E_T = 10^{15}$ eV trajectories.

For an $E_T = 10^{15}$ eV primary ion that delivers $E_C = 10^{20}$ eV of energy to the atmosphere, E_T is insignificant compared to E_C and (1) gives

$$E_0^{n-1} = 10^{15n-20} \quad , \tag{2}$$

for E_0 measured in eV. This determines the threshold energy E_0 as a function of n.

For the low energy end, the 1 TeV protons at Fermilab have been well-studied and no effect has been reported. Then, for $E_T = 10^{12}$ eV = 1 TeV, put the proposed effect at less than 10%, i.e. $(E_C - E_T)/E_T \leq 10^{-1}$ (= 10%). One finds that

$$10^{(12+\frac{1}{n-1})} \text{ eV} \quad \leq E_0 \quad . \tag{3}$$

The inequality shows that, for reasonable values of n, these assumptions put the threshold E_0 at about 1 TeV or more.

Combining (2) and (3), we have
$$3 \leq n \quad . \tag{4}$$
Given the range in energies proposed in the literature for the 'highest energy SNR-accelerated ions' and the ambiguity in what energy to assign to the 'highest energy cosmic rays', the choice of 10^{15} eV for one and 10^{20} eV for the other is in part motivated by the result that the minimum n is integral. Other reasonable choices give fractional values near three for the minimum n.

By (2), we get
$$E_0 = 3 \times 10^{12} \times 10^{\frac{5(n-3)}{2(n-1)}} \text{ eV} \quad . \tag{5}$$
Every increase in n by one multiplies E_0 by a factor of $10^{5/[n(n-1)]}$.

To be definite, suppose $n = 3$. Then we have
$$E_C = E_T + 10^{-25}E_T^3 \quad , \tag{6}$$
for energies measured in eV. Such a dependence, with $n = 3$ and 4.8×10^{-26} in place of 10^{-25}, has been deduced from considerations based on quantum field theory and spacetime symmetries.[7]

The existence of the effect and the values of n and E_0 have yet to be determined by experiment. At CERN operators have recently begun running the new accelerator, the Large Hadron Collider (LHC).[2] The LHC has design energies of 7 TeV, nearly a factor of ten higher than the 1 TeV used in the above calculation. The 'less than 10% effect' with 1 TeV protons that is too small to notice becomes a 'less than $7^{n-1} \cdot 10\%$ effect' with 7 TeV protons.

With $n = 3$, a 2% effect at 1 Tev becomes a $7^2 \cdot 2\% = 100\%$ effect at 7 TeV. This means a proton with a 7 TeV trajectory delivers up to 14 TeV in a collision. The LHC is a proton anti-proton collider so an $E_T = 7$ TeV trajectory energy proton colliding head-on with an $E_T = 7$ TeV trajectory energy anti-proton delivers, not the design $2E_T = 14$ TeV, but a $2E_C = 28$ TeV collision.

If the effect is real and the threshold for the effect with protons is approximately 1 TeV, then the collision energies E_C should be significantly greater than the trajectory energies E_T at the LHC.

References

[1] Fermi National Accelerator Laboratory, Tevatron Department, current online web page, http://www-bdnew.fnal.gov/tevatron/ .

[2] The Large Hadron Collider Project (http://lhc.web.cern.ch/lhc/).

[3] See, for example, Y. Butt, *Beyond the myth of the supernova-remnant origin of cosmic rays*, Nature, Vol 460, August 2009, jdoi:10.1038/nature08127, pp. 701-704

[4] See, for example, M. Nagano and A. A. Watson, *Observations and implications of the ultrahigh-energy cosmic rays*, Rev. Mod. Phys., Vol. 72, No. 3, July 2000, pp. 689-732

[5] See, for example,J. Blümer, R. Engel, J. Hörandel, *Cosmic rays from the knee to the highest energies,* Prog. Part. Nucl. Phys. 63 (2009) 293.

[6] See, for example, J. L. Han, *Magnetic fields in our Galaxy: How much do we know? III. Progress in the last decade*, Chin. J. Astron. Astrophys. Vol. 6, 2006, pp. 211-217

[7] R. Shurtleff, *Dual Phase Cosmic Rays*, http://lanl.arxiv.org/abs/0801.0071 , on-line physics arXiv, 2008.

A   Exercises

Supplemental exercises are provided for the online version. Answers can be found as remarks coded by % in the LaTeX source which can be downloaded from the arXiv.

1. Find the threshold energy E_0 for $n = 3$ and a 2% effect for an ion with a trajectory energy of 1 TeV.

2. Rewrite (4) for a proton with trajectory energy $E_T = 10^a mc^2$ that delivers a collision energy $E_C = 10^b mc^2$. Assume the effect is a 10% difference at an energy $E_T = 10^c mc^2$. Use $mc^2 = 10^9$ eV as the proton rest energy, $a, b, c \geq 0$, $a - c > 0$, and $10^{b-a} \gg 1$. (The values used in the text are $a = 6$, $b = 11$, and $c = 3$.)

3. Cosmic ray energies are collision energies E_C while the energies of ions accelerated at the SNR are trajectory energies E_T. Given that the observed cosmic ray spectrum at high energies is roughly proportional to the inverse cube of E_C, $dN/dE_C \propto E_C^{-3}$, i.e. a spectral index of -3, determine the spectral index of dN/dE_T at an SNR shockwave.